TITLE: The Problem of Orthogonality of Eigenwaves in a Waveguide Partially Filled with a Lossy Dielectric

THE PROBLEM OF ORTHOGONALITY OF EIGENWAVES IN A WAVEGUIDE PARTIALLY FILLED WITH A LOSSY DIELECTRIC

L.P.Yatsuk, A.A.Komyachko, Yu.V.Zhironkin

V.N.Karazin Kharkov National University, 4, Svobody Sq., Kharkov, 61077, Ukraine

Tel. (0572)45-75-48, Email: Ludmila.P.Yatsuk@univer.kharkov.ua

The eigenwaves orthogonality is investigated for the rectangular waveguide having a lossy dielectric slab inside it. It is shown analytically that eigenwaves of such waveguides are energetically non-orthogonal. The interaction of pair of non-orthogonal eigenwaves is discussed. The numerical estimation of their non-orthogonality has been carried out for the various slab dimensions, its permittivity and losses. Calculation results are presented.

INTRODUCTION

The property of eigenwaves orthogonality is very important one while solving scattering problems. According to [1] there are two definitions of the orthogonality:

Mathematical orthogonality

$$\frac{1}{2}\int_{S_\perp}\left[\vec{E}_\mu,\vec{H}_\nu\right]\vec{z}^{\,0}dS=\delta_{\mu\nu}N_\mu \tag{1}$$

and energetic one

$$\frac{1}{2}\int_{S_\perp}\left[\vec{E}_\mu,\vec{H}^*_\nu\right]\vec{z}^{\,0}dS=\delta_{\mu\nu}P_\mu . \tag{2}$$

Here δ is a Kronecker symbol, N_μ is a norm, P_μ is a longitudinal μ - mode complex flux of power. The equality (1) is fulfilled always. As to the equality (2), it has been shown in [1], that the pairs of complex waves with propagation constants $\pm\gamma_\mu$ and $\pm\gamma^*_\mu$ are energetically non-orthogonal. Their joint existence results in arising active and reactive power fluxes over a waveguide cross section. We have shown that not only these waves but the modes with $\gamma_\mu\neq\gamma_\nu$ in a waveguide filled with a layered dissipative dielectric are energetically non- orthogonal.

SOME POINTS OF THEORY

The rectangular waveguide of $a\times b$ cross section with a lossy dielectric slab of $a_2\times b$ cross section (Fig.1) is considered.. The $\vec{E}$ and $\vec{H}$ fields of *LE*- and *LM*-modes modes can be found using the magnetic Hertz vector $\vec{\Pi}^m=\vec{x}^0\Pi^m_x$ and electric one $\vec{\Pi}^e=\vec{x}^0\Pi^e_x$ correspondingly. Components Π^m_x and Π^e_x can be represented as

$$\Pi_x^{m(e)}(x,y,z)=\varphi^{m(e)}(x)f^{m(e)}(y)e^{\mp i\gamma_{\mu n}^{m(e)}z}, \tag{3}$$

where $\varphi^m(x)$, $\varphi^e(x)$ – scalar functions which satisfy the following equation

$$\frac{d^2\varphi_\mu^{m(e)}(x)}{dx^2}+\left(\alpha_\mu^{m(e)}\right)^2\varphi_\mu^{m(e)}(x)=0 \tag{4}$$

and corresponding boundary conditions on the metal walls and dielectric interfaces. It is easy to show from (4) that for two solutions with μ and ν numbers of the dispersion equation root it is easy to obtain the following result.

$$\Phi_{\mu\nu}=\int_0^a\varphi_\mu^{m(e)}(x)\cdot\left(\varphi_\nu^{m(e)}(x)\right)^* dx=\frac{-i2k^2\varepsilon''}{\left(\gamma_\mu^{m(e)}\right)^2-\left(\left(\gamma_\nu^{m(e)}\right)^2\right)^*}\int_{x_1}^{x_2}\varphi_\mu^{m(e)}(x)\cdot\left(\varphi_\nu^{m(e)}(x)\right)^* dx\,. \tag{5}$$

If $\varepsilon''\neq 0$ and $x_2-x_1\neq a$ the right side of (7) differs from zero. It proves, that scalar functions $\varphi_\mu^{m(e)}(x)$ and $\varphi_\nu^{m(e)}(x)$ are energetically non-orthogonal. The real part of the integral (2) for *LE*-modes with indices μ and ν takes the following form:

$$\begin{aligned}\operatorname{Re}P_{\mu\nu}&=\frac{1}{2}\operatorname{Re}\int_{S_\perp}\left[\vec{E}_\mu^{LE},\left(\vec{H}_\nu^{LE}\right)^*\right]\vec{z}^0 dS=\\&=-\frac{\omega\mu_0 b}{2(2-\delta_{0n})}\operatorname{Re}\left\{\gamma_\mu^m\left(\kappa_n^2+\left(\gamma_\nu^m\right)^2\right)^* e^{-i\left(\gamma_m^m-\left(\gamma_n^m\right)^*\right)z}\int_{S_\perp}\varphi_\mu^m(x)\left(\varphi_\nu^m(x)\right)^* dS\right\}\end{aligned} \tag{6}$$

If the electromagnetic field in a waveguide consists of two eigen waves fields $\vec{E}=\vec{E}_\mu+\vec{E}_\nu$, $\vec{H}=\vec{H}_\mu+\vec{H}_\nu$ (each wave of unit amplitude for simplicity), the total flux of energy of this field $P=\frac{1}{2}\operatorname{Re}\int_{S_\perp}\left[\vec{E},\vec{H}^*\right]\vec{z}^0 dS$ consists of four fluxes:

$$P=P_{\mu\mu}+P_{\nu\nu}+P_{\mu\nu}+P_{\nu\mu}\,. \tag{7}$$

Two of them ($P_{\mu\mu}$ and $P_{\nu\nu}$) may be named eigen power fluxes of the waves with indexes μ and ν. The two others ($P_{\mu\nu}$ and $P_{\nu\mu}$) – mutual fluxes caused by field combinations of various modes: $\left[\vec{E}_\mu,\vec{H}_\nu^*\right]$ and $\left[\vec{E}_\nu,\vec{H}_\mu^*\right]$.

The eigenwaves $\vec{E}_\mu,\vec{H}_\mu$ and $\vec{E}_\nu,\vec{H}_\nu$ propagates with different velocities. Therefore a phase shift between $\vec{E}_\mu$ and $\vec{H}_\nu$ ($\vec{E}_\nu$ and $\vec{H}_\mu$ as well) changes along *z*-axis. As an effect so does the phase of the complex vectors $\left[\vec{E}_\mu,\vec{H}_\nu^*\right]$ and $\left[\vec{E}_\nu,\vec{H}_\mu^*\right]$. It causes variation not only magnitudes of the real part of these vectors but their sign as well. So, due to the interference effect, the mutual fluxes $P_{\mu\nu}$ and $P_{\nu\mu}$ depending on z may increase or decrease the total flux in comparison with $P_{\mu\mu}+P_{\nu\nu}$. This process is analogues to the vector addition on the phase plane. The square length of the sum vector according to the cosine theorem may be equal, less or greater than sum of squared lengths of vectors, which are summarized. As an effect of this the total flux of energy in the waveguide cross section oscillates depending on *z* around the exponentional curve, representing the sum $P_{\mu\mu}+P_{\nu\nu}$ (Fig.4)

CALCULATION RESULTS

Calculations were carried out for the waveguide 23x10 mm^2, various values of ε, $tg\delta$ and dimensions of dielectric slab. It was determined, that the orthogonality of eigenwaves can be confirmed only if the eigenvalues are determined with high accuracy (10^{-12}-10^{-15}). The value of the integral $\Phi_{\mu\nu}$ (5) is represented on the Fig.2. Along the μ,ν-axis root numbers change, along x-one –width of the slab, $\varepsilon = 10$, $tg\delta = 0{,}1$. The Fig.3 represents $P_{\mu\nu}$ and $P_{\nu\mu}$, when $\mu = 1$, $\nu = 3$, $\varepsilon = 15$, $tg\delta = 0{,}1$. The same parameters were used for the Fig.4.

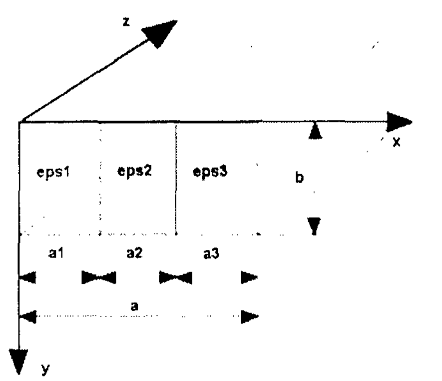

Fig.1

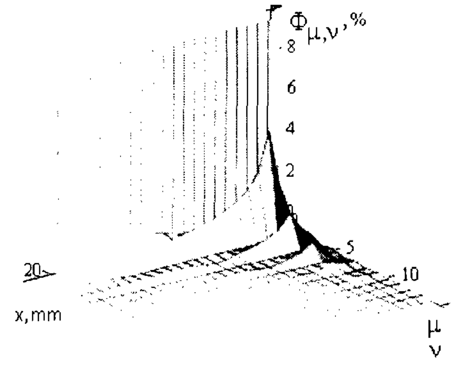

Fig. 2

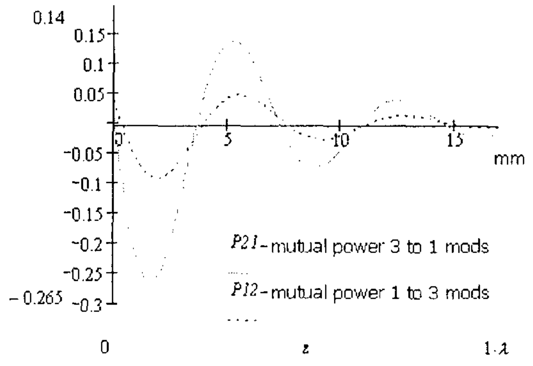

Fig. 3

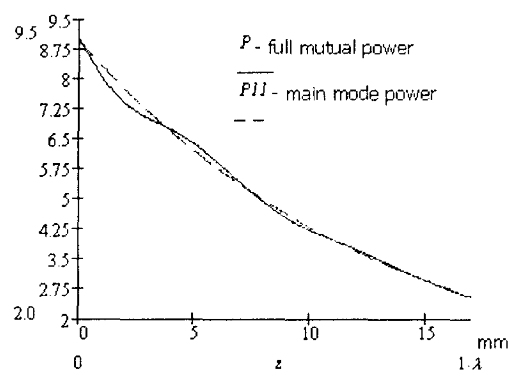

Fig. 4

CONCLUSION

It has been shown that the energetic non-orthogonality of the eigenwaves is the more noticeable, the higher the ε and $tg\delta$ of dielectric is. By $\varepsilon < 5$, $tg\delta < 0{,}01$ it can be neglected. The non-orthogonality effect diminishes with rising slab width and vanishes at all when the lossy dielectric fully fills the waveguide.

REFERENCES

[1] Veselov G.I., Rayevski S.B. The Layered Metal-Dielectric waveguides.-M.: "Radio and Communication".-1988, 247 p. (in Russian).